Evolutionary Algorithms Winter 2017/2018
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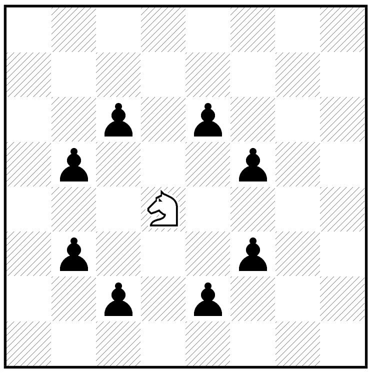
Exercise Sheet 3

Exercise 9 The Knight's Tour Problem



The knight's tour problem consists in the task to find a tour of a knight (a piece in the game of chess) across an $n \times n$ chess board, such that each square is visited exactly once. (One may introduce as an additional requirement, which makes the task more difficult, that the knight should be able to return from the last square to the initial square, that is, that the knight's tour is a Hamiltonian cycle.) The squares a knight may move to are indicated on the chessboard on the right: they are marked by black pawns (also pieces in the game of chess).

How can one approach the knight's tour problem with backtracking?
Why is this approach not advisable?

Exercise 10 The Knight's Tour Problem

a) How can the knight's tour problem (see Exercise 9) be approached with hill climbing or simulated annealing? In particular, specify how a candidate tour is evaluated in these approaches!

b) Which operations could be used to obtain a (small) random variation of a given candidate tour? How can one implement the search algorithm efficiently if these operations are used?

Exercise 11 Four Color Problem

The four color problem is one of the most famous problems of mathematics. It consists in the question whether any map of countries can be colored with at most four colors such that no two states that share a border are colored with the same color. This problem, which was posed first by Francis Guthrie in 1852 (first published 1878), was unsolved for a long time. (Note that the problem makes the implicit assumption, that all countries are connected areas, that is, that they do not consist of two or more unconnected territories.) Only in 1976 Wolfgang Haken and Kenneth Appel were able to prove the four color theorem with the help of an extensive computer program. Because of the high complexity of the program and the long computation time, many mathematicians are very skeptical about the result.

a) Consider a (political) map of central europe with the states Belgium, Germany, France, Luxemburg, Netherlands, Austria, Poland, Switzerland, Slovakia, Czech Republic and Hungary. How can one find a coloring of this map that satisfies the four color theorem with the help of an evolutionary algorithm?

b) Generalize the approach developed in part a) to arbitrary graph coloring problems! (That is, the task to find a coloring of the vertices of a graph such that no two vertices that are connected by an edge bear the same color.)

Exercise 12 Gray Codes

Using the method discussed in the lecture, compute the Gray codes for

a) the number 0.7 in the interval $[0, 2]$ with a representational precision of 10^{-4},
b) the number 18.6 in the interval $[12, 21]$ with a representational precision of 10^{-2},
c) the number 0 in the interval $[-2, 1]$ with a representational precision of 10^{-3}.

Why are Gray codes used in evolutionary algorithms?