

### Organisational Rules for Exam Admission

The lecture is accompanied by exercise sheets. At the beginning of each exercise lesson, the exercises are *voted for*. By voting for an exercise, one expresses one's willingness to present something about it. (Suggestions for a solution will be discussed, they need not be correct right away.) To the exam will be admitted who

1. voted for *at least* half of the exercises *and*
2. presented something for *at least* two exercises.

### Exercise Sheet 1 — Reminder: Vector Computations and Geometry

#### Exercise 1 Vector Computations

Let the two vectors  $\vec{x} = (3, 1, 2)^\top$  and  $\vec{y} = (2, 3, 4)^\top$  be given. Compute

- a) the difference vector pointing from  $\vec{x}$  to  $\vec{y}$ ,
- b) the scalar (or inner) product  $\vec{x}^\top \cdot \vec{y}$ ,
- c) the vector product  $\vec{x} \times \vec{y}$ .
- d) the matrix (or outer) product  $\vec{x} \cdot \vec{y}^\top$ ,
- e) the angle between the two vectors.

#### Exercise 2 Hessian Normal Form

Transform the following line equations into Hessian normal form  $\vec{r}^\top \cdot \vec{n}_0 - d = 0$ :

- a)  $3x - 4y - 20 = 0$ ,
- b)  $x + y + 3 = 0$ ,
- c)  $y = bx + c$  mit  $c < 0$ .

#### Exercise 3 Distances to Straight Lines

- a) What distance does the origin have from the line  $12x - 5y + 39 = 0$ ?
- b) What distance does  $P_1 \hat{=} (4, 3)^\top$  have from the line that intersects the coordinate axes at  $x = \frac{10}{3}$  and  $y = \frac{5}{2}$ ?
- c) What distance do the parallels  $2x - 3y = 6$  and  $4x - 6y = 25$  have?

#### Exercise 4 Plane Equations

Which plane through the point  $\vec{p} = (-3, 0, 2)^\top$  is perpendicular to the line  $\vec{x} = (-1, -2, 0)^\top + k \cdot (1, 1, -1)^\top$  for  $k \in \mathbb{R}$ ?

**Exercise 5** Points on a Plane

A plane through the point with location vector  $\vec{p}$  that is perpendicular to the vector  $\vec{n}$  has the equation  $(\vec{x} - \vec{p})^\top \vec{n} = 0$ . Let  $\vec{p} = (1, -1, 2)^\top$  and  $\vec{n} = (1, 2, -3)^\top$ .

Which of the following points lie in the plane?

$P_1 \hat{=} (-2, -1, 1)^\top$ ,  $P_2 \hat{=} (1, -1, 2)^\top$ ,  $P_3 \hat{=} (2, -2, 1)^\top$ .

**Exercise 6** Distance of a Point from a Plane

Find a formula to compute the distance of a given point  $\vec{y} = (y_1, y_2, y_3)^\top$  to a plane  $(\vec{x} - \vec{p})^\top \vec{n} = 0$ . Use this formula to compute the distance for  $\vec{p} = (-1, -1, -1)^\top$ ,  $\vec{n} = (4, -2, 3)^\top$  and  $\vec{y} = (3, 14, -6)^\top$ .