Exercise Sheet 2

Exercise 7 Threshold Logic Units

Determine the parameters of *single* threshold logic units in such a way that these units compute the following Boolean functions:

a) $y = x_1 \vee x_2$
b) $y = \neg x_1 \wedge x_2$

(Hint: A threshold logic unit computes on which side of a (hyper-)plane a given input vector lies. Start from a geometric representation.)

Exercise 8 Threshold Logic Units

Determine the parameters of *single* threshold logic units in such a way that they compute the following Boolean functions:

a) $y = x_1 \wedge \neg x_2 \wedge x_3$ (or short: $x_1\overline{x_2}x_3$)
b) $y = (x_1 \wedge \neg x_2) \vee (x_1 \wedge x_3)$ (or short: $x_1\overline{x_2} \vee x_1x_3$)
c) $y = (x_1 \wedge x_2) \vee (\neg x_2 \wedge x_3)$ (or short: $x_1x_2 \vee \overline{x_2}x_3$)
d) $y = (x_1 \wedge x_2) \vee \neg x_3$ (or short: $x_1x_2 \vee \overline{x_3}$)

(Hint: A threshold logic unit computes on which side of a (hyper-)plane a given input vector lies. Start from a geometric representation.)

Exercise 9 Threshold Logic Units

Try to find parameters of a threshold logic unit in such a way that it computes the Exclusive Or (written $x_1 \dot{\vee} x_2$ or $x_1 \oplus x_2$)! What problem does one run into? How can one solve this problem? (Hint: Recall the solution of the biimplication problem that was studied in the lecture.)

Exercise 10 Networks of Threshold Logic Units

Construct a neural network of threshold logic units that produces the output 1 for points (x_1, x_2) inside of the triangle shown in the sketch and the output 0 for points outside!

(Hint: Recall the neural network for the biimplication problem that was studied in the lecture and interpret the computations of the threshold logic units in the first layer as a coordinate transformation.)