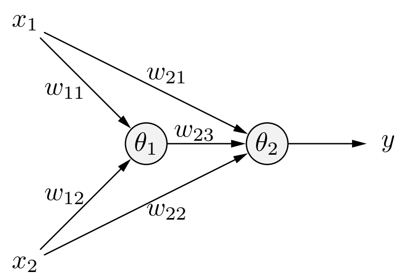
Artificial Neural Networks and Deep Learning Summer 2018
Christian Borgelt and Christoph Doell from 2018.05.15

Exercise Sheet 3

Exercise 11 Networks of Threshold Logic Units



Determine the parameters w_{ji} and θ_j of the neural network of threshold logic units that is shown in the sketch on the right in such a way that it computes the Exclusive Or of the Boolean variables x_1 and x_2 (that is, $y = x_1 \dot{\vee} x_2$ or $y = x_1 \oplus x_2$)!

(Hint: Start from a geometric interpretation of the computation in the input space of the neuron on the right and consider how you may use the output of the left neuron to arrange the points (x_1, x_2), for which 1 or 0 should be produced, in such a way that they become separable by a plane.)

Exercise 12 Threshold Logic Units: Representation of Boolean Functions

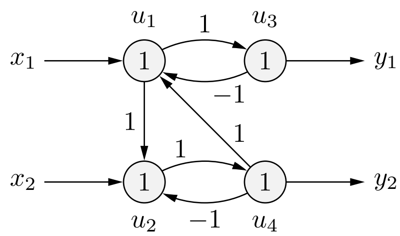
In the lecture we studied an algorithm with which, given an arbitrary Boolean function $y = f(x_1, \ldots, x_n)$, a network of threshold logic units could be constructed that computed this function. This algorithm was based on representing the Boolean function in disjunctive normal form. Describe a dual algorithm that relies on representing the Boolean function in *conjunctive* normal form!

Exercise 13 Training Threshold Logic Units

Show how a theshold logic unit is trained (with the delta rule) to compute the Boolean function $x_1 \to x_2$! (Preferably with the help of a table that has columns for the values of x_1, x_2, $o = x_1 \to x_2$, $\vec{x} \cdot \vec{w}$, y, e, $\Delta\theta$, Δw_1, Δw_2, θ, w_1 and w_2.) Start from the initial state $\vec{w} = (0, 0, 0)$ of the (extended) weight vector and use 1 as the learning rate. Interpret the learning result geometrically!

Exercise 14 Update Order

Consider the following network of threshold logic units:



Show that it depends on the update order of the threshold logic units whether the network reaches a stable state if the the inputs $x_1 = 0$ and $x_2 = 1$ are presented!