**Artificial Neural Networks and Deep Learning** Summer 2018
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# Exercise Sheet 4

## Exercise 15 Method of Least Squares/Regression

Determine the best fit straight line $y = a + bx$ (regression line) for the following two data sets with the help of the method of least squares:

a) $(-2, 0), (0, 1), (1, 3), (2, 5)$
b) $(-1, 3), (1, 2), (2, 0), (4, -2)$

Draw the data points and the best fit straight line!

## Exercise 16 Method of Least Squares/Regression

Determine and draw a best fit parabola $y = a + bx + cx^2$ for the data of Exercise 15a!

## Exercise 17 Exponential Regression

Radioactive substances decay according to the law $N(t) = N_0 e^{-\lambda t}$, where $t$ is the time, $\lambda$ is a substance-specific decay parameter, $N_0$ is the number of particles at the beginning and $N(t)$ the remaining number of particles of the substance at time $t$. With the help of a Geiger-Müller counter the following numbers $n$ of $\alpha$-particles, which are emitted by a small probe of a radioactive substance, are counted up to points $t$ in time:

| $t$ (in s) | 0 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 |
|---|---|---|---|---|---|---|---|---|---|
| $n$ | 0 | 306 | 552 | 655 | 768 | 863 | 901 | 919 | 956 |

Each counted $\alpha$-particle indicates the decay of one particle of the radioactive substance. Determine the half-life of the radioactive substance!

Procedure: Determine a best fit curve $n = n_0(1 - e^{a+bt})$ for the given data points! (Hint: You have to find a transformation that reduces the problem to determining a regression line; $n_0 = 1000$.) Even though this yields a value for $a$ that differs from 0, one may reasonably see $-b$ as an approximation of the decay parameter $\lambda$, from which the half-life can be computed.

## Exercise 18 Logistic Regression

The following table shows the number of American intercontinental ballistic missiles (ICBMs) in the 1960s:

| year, $x$ | 1960 | 1961 | 1962 | 1963 | 1964 | 1965 | 1966 | 1967 | 1968 | 1969 |
|---|---|---|---|---|---|---|---|---|---|---|
| number, $y$ | 18 | 63 | 294 | 424 | 834 | 854 | 904 | 1054 | 1054 | 1054 |

Find a best fit curve with the help of logistic regression ($Y = 1060$)!
Draw the original data and sketch the curve $y = \frac{1060}{1+e^{-(a+bx)}}$!