Artificial Neural Networks and Deep Learning — Summer 2018
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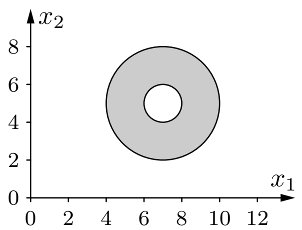
Exercise Sheet 9

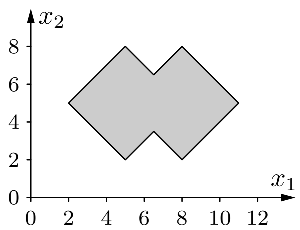
Exercise 35 — Radial Basis Function Networks: Binary Functions

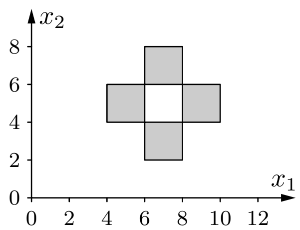
Determine the parameters (weights $\vec{w}_u$ and radii σ_u) of radial basis function networks with the activation function

$$f_{\text{act}}^{(u)}(\text{net}_u, \sigma_u) = \begin{cases} 1, & \text{if } \text{net}_u \leq \sigma_u, \\ 0, & \text{otherwise,} \end{cases}$$

for the neurons in the hidden layer, that produce the value 1 for points inside the gray areas of the diagrams shown below and the value 0 outside! It does not matter whether the networks produce a value of 0 or a value of 1 for points on the boundaries of the gray areas. However, you should make sure that for eavery point in the x_1-x_2 plane *either* a value of 0 *or* a value of 1 is computed.

a)


b)


c)


Exercise 36 — Radial Basis Function Networks: Function Approximation

a) Construct a radial basis function network with about 10 neurons that approximates the function $y = x^2$ in the interval $[0.5, 4.5]$ by a step function!

b) How can the approximation be improved? (State at least two possibilities.)

Exercise 37 — Radial Basis Function Networks: Initialization

Using the method of the (Moore–Penrose) pseudo-inverse, determine the parameters (weights $\vec{w}_u$ and bias values θ_u) of radial basis function networks that compute the conjunction $x_1 \wedge x_2$! Employ

a) two radial basis functions mit centers $(0, 0)$ and $(1, 1)$,
b) one radial basis function with center $(1, 1)$.

All basis functions should have the radius $\frac{1}{2}$. The hidden neurons should have the Euclidean distance as their network input function and a Gaussian function

$$f_{\text{act}}(\text{net}_u, \sigma_u) = e^{-\frac{\text{net}_u^2}{2\sigma_u^2}}$$

as their activation function. Compute the actual output of the two networks and compare it to the desired outputs! Why do we obtain a perfect solution of the learning problem in part a)?

Exercise 38 Radial Basis Function Networks: Initialization

Using the method of the (Moore–Penrose) pseudo-inverse, determine the parameters (weights $\vec{w}_u$ and bias values θ_u) of radial basis function networks that compute the Exclusive Or $x_1 \dot{\vee} x_2$ (or $x_1 \oplus x_2$)! Employ

a) two radial basis functions mit centers $(0,0)$ and $(1,1)$,
b) one radial basis function with center $(1,1)$.

All basis functions should have the radius $\frac{5}{4}$. The hidden neurons should have the city block distance (also known as Manhattan distance) as their network input function and a triangular function as their activation function. Compute the actual output of the two networks and compare it to the desired outputs!