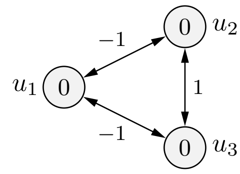
Artificial Neural Networks and Deep Learning Summer 2018
Christian Borgelt and Christoph Doell from 2018.07.17

Exercise Sheet 11

Exercise 43 Hopfield Networks: State Graph

The diagram on the right shows a simple Hopfield network. For this network, determine the final state(s) that are reached starting from the initial state $(\text{act}_{u_1}, \text{act}_{u_2}, \text{act}_{u_3}) = (-1, -1, -1)$!
(Hint: Use a state transition graph. In this graph, mark the initial state and the final state(s). Notice that there is no explicit start neuron and that therefore you have to consider all possible subsequent states.)



Exercise 44 Hopfield-Netze: Energy Function

Determine the energy function of the Hopfield network of Exercise 43! With the help of this energy function, compute the energies of the individual states of the network. Then arrange the states according to their energy, that is, draw a new state transition graph, in which the location of the states indicates their energy!

Exercise 45 Hopfield Networks: Pattern Recognition

We want to store the two patterns $(-1, +1, -1, +1)$ and $(+1, -1, -1, +1)$ in a Hopfield network with four neurons, that is, these two patterns are supposed to be the stable states of the network, which cannot be left, regardless of which neuron is updated.

a) Compute the connection weights and the threshold values of the neurons of a Hopfield network that stores the abovementioned patterns!

b) How many other patterns can be stored in this network?

c) Find two more patterns that could be stored in the network constructed in part a) without "forgetting" the old patterns! (Of course, in order to actually store these patterns, the connection weights may have to be modified.)

Exercise 46 Hopfield Networks: Solving Optimization Problems

We are given a sequence $F = (a_1, a_2, \ldots, a_n)$ of integer numbers. For simplicity, we assume that at least one of these numbers is non-negative. We desire to know the maximum partial sum of this sequence, that is, the maximum of sums of partial sequences of the sequence F, where by a partial sequence of the sequence F we mean a sequence $F_{ij} = (a_i, a_{i+1}, \ldots, a_j)$ with $1 \leq i \leq j \leq n$. That is, we desire to know

$$\text{mts}(F) = \max_{1 \leq i \leq j \leq n} \sum_{k=i}^{j} a_k.$$

Construct a Hopfield network to solve this optimization problem!
(Hint: You have to find a suitable energy function.)